Red Level

1) **(5 credits)** Sheldon Cooper read his 10 digit mobile phone number aloud to his friends. Howard Wolowitz and Raj Koothrappali failed to write down some digits: Howard got '8248312', and Raj got '38128432'. Restore Sheldon's phone number [taking into account that each digit is repeated in his number at most two times]. (Please find all alternative solutions) PLEASE CONSIDER TO WHAT EXTENT THE CONDITION IN SQUARE BRACKETS IS USEFUL TO SIMPLIFY THE EXPLANATIONS (it does not affect the answer)

2) **(6 credits)** Forty thieves filled one of their chests with gold and silver dust to the top; the quantity of gold dust was 2 times more than the quantity of silver dust. According to calculations of Ali Baba, after pouring one-half of the quantity of silver dust out of the chest and filling the chest with gold dust to the top, the cost of dust in this chest will increase by 20 percent. By what percentage will the cost of dust in the chest be decreased if Ali Baba pours one-half of the quantity of gold dust out of the chest and fills the chest with silver dust to the top?

3) **(8 credits)** Each of brothers Grimm buys a pastry or a tub of ice cream every day. The younger brother always buys the meal bought by his older brother a week ago, and the older brother never buys the meal bought by his younger brother a week ago. What is the maximum number of tubs of ice cream that could be bought by the older brother in November?

4) **(8 credits)** In the triangle ABC, the measure of the angle C is three times greater than the measure of the angle A, and the side AB is twice as long as the side BC. Find the measure of the angle B.

5) **(10 credits)** 150 employees (in total) work in a two-storey office. When each male office employee wrote a message to a female employee working on another floor, thirty female office employees did not receive any message, and each of other female employees received one message. At the same time, the number of those first-floor female employees, who received the message, is less by half than the total number of employees on the first floor, and the number of those second-floor female employees, who received the message, is equal to 2/7 of the total number of employees on the second floor. How many male employees work on the second floor?

6) **(10 credits)** $x + y + z = 0$. What is the largest number of positive numbers among $\sin(x)$, $\sin(y)$, $\sin(z)$, $\cos(x)$, $\cos(y)$, $\cos(z)$?

7) **(12 credits)** Artemis Fowl selected four natural numbers. Then he calculated the product of the first, second and fourth numbers, the product of the first, third and fourth numbers, the product of the second, third and fourth numbers and finally the sum of the first, second and third numbers. After that, he wrote the results on the board in the ascending order: 24, 27, 120, 160. Restore the numbers selected by Artemis in the correct order.

8) **(12 credits)** Ulysses moved from the point A to the point B along the polygonal chain ACB consisting of 2 segments. When moving along it, Ulysses constantly moves away from the point A and becomes closer to the point B. Any longer polygonal chain of 2 segments connecting the points A and B does not meet such condition. Find the measure of the angle ACB.

9) **(14 credits)** Scrooge McDuck has 19 coins: 18 identical real coins and 1 fake coin (its weight is a little less than the weight of a real coin). He also has 2 two-pan scales; each of them can show the bowl on which coins are heavier, but in this case it breaks due to imbalance. Scrooge has developed an algorithm which makes it possible to find a fake coin in three weighings, even if such weighings result in the breakage of the scales. How many coins should he put on the bowls (such action is the first stage of his algorithm)? [Apparently, the uniqueness of this solution does not need to be proved.]

10) **(15 credits)** Each cell of the 100 x 100 board contains a natural number, and all such numbers are different. The White King wants to pass several squares of the board (he chooses the starting point at his discretion) provided that the number contained in each next square is greater than the number contained in the previous one. What is the maximum number of squares that can be visited by the King for sure (regardless of the arrangement of the numbers)?